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A Level Further Mathematics B (MEI)
Y421 Mechanics Major
Sample Question Paper

Model Answers.

Date – Morning/Afternoon
Time allowed: 2 hours 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet.
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is 120.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of 20 pages. The Question Paper consists of 12 pages.

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Section A (26 marks)

Answer all the questions

1 A particle P has position vector r m at time t s given by $r = (t^3 - 3t^2)i - (4t^2 + 1)j$ for $t \geq 0$. Find the magnitude of the acceleration of P when $t = 2$. [4]

$$v = \frac{dr}{dt} = (3t^2 - 6t)\underline{i} - (8t)\underline{j}$$

$$a = \frac{dv}{dt} = (6t - 6)\underline{i} - (8)\underline{j}$$

$$t = 2 \Rightarrow a = 6\underline{i} - 8\underline{j}$$

$$|a| = \sqrt{6^2 + (-8)^2}$$

$$= \sqrt{100}$$

$$= 10\ ms^{-2}$$

2 A particle of mass 5 kg is moving with velocity $2i + 5j$ m s^{-1}. It receives an impulse of magnitude 15 N s in the direction $i + 2j - 2k$. Find the velocity of the particle immediately afterwards. [3]

Unit vector of $\underline{i} + 2\underline{j} - 2\underline{k}$: $\dfrac{\underline{i} + 2\underline{j} - 2\underline{k}}{\sqrt{1+4+4}}$

$$= \frac{1}{3}(\underline{i} + 2\underline{j} - 2\underline{k})$$

$$I = 15\,Ns \Rightarrow I = 15 \times \frac{1}{3}(\underline{i} + 2\underline{j} - 2\underline{k}) = 5(\underline{i} + 2\underline{j} - 2\underline{k})$$

$$I = mv - mu$$

$$\Rightarrow 5(\underline{i} + 2\underline{j} - 2\underline{k}) = 5(v - (2\underline{i} + 5\underline{j}))$$

$$\Rightarrow \underline{i} + 2\underline{j} - 2\underline{k} = v - 2\underline{i} - 5\underline{j}$$

$$\Rightarrow v = (3\underline{i} + 7\underline{j} - 2\underline{k})\ ms^{-1}$$

3 The fixed points E and F are on the same horizontal level with EF = 1.6 m. A light string has natural length 0.7 m and modulus of elasticity 29.4 N. One end of the string is attached to E and the other end is attached to a particle of mass M kg. A second string, identical to the first, has one end attached to F and the other end attached to the particle. The system is in equilibrium in a vertical plane with each string stretched to a length of 1 m, as shown in Fig. 3.

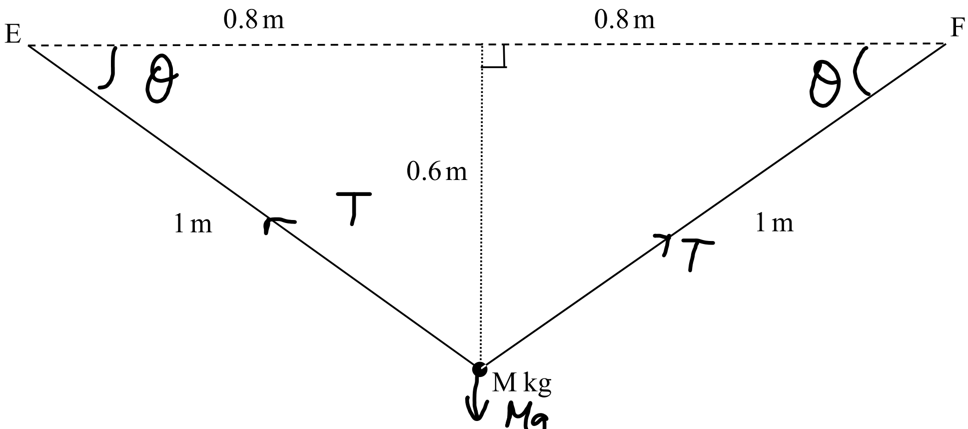


Fig. 3

(i) Find the tension in each string. [2]

Hoke's law: $T = \dfrac{\lambda x}{l_0}$

$$= \frac{29.4 \times (1 - 0.7)}{0.7}$$

$$= \underline{\underline{12.6\ \text{N}}}$$

(ii) Find M. [3]

Resolving vertically: $2T\sin\theta = Mg$

$$\Rightarrow 2 \times 12.6 \times \frac{0.6}{1} = Mg$$

$$\Rightarrow M = \frac{2 \times 12.6 \times 0.6}{g}$$

$$\Rightarrow M = \underline{\underline{1.54\ \text{kg}}}$$

4 A fixed smooth sphere has centre O and radius a. A particle P of mass m is placed at the highest point of the sphere and given an initial horizontal speed u.

For the first part of its motion, P remains in contact with the sphere and has speed v when OP makes an angle θ with the upward vertical. This is shown in Fig. 4.

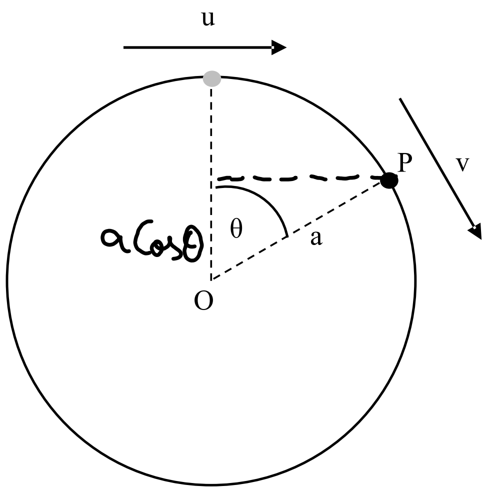


Fig. 4

(i) By considering the energy of P, show that $v^2 = u^2 + 2ga(1 - \cos\theta)$. [2]

Conservation of Energy : Initial KE + Initial PE = Final KE + Final PE

$\Rightarrow \frac{1}{2}mu^2 + mgh_0 = \frac{1}{2}mv^2 + mgh_1$

$\Rightarrow \frac{1}{2}u^2 + ga = \frac{1}{2}v^2 + ga\cos\theta \Rightarrow u^2 + 2ga = v^2 + 2ga\cos\theta$

$\Rightarrow v^2 = u^2 + 2ga - 2ga\cos\theta \Rightarrow \underline{\underline{v^2 = u^2 + 2ga(1-\cos\theta)}}$

(ii) Show that the magnitude of the normal contact force between the sphere and particle P is

$mg(3\cos\theta - 2) - \frac{mu^2}{a}$. [2]

R, g $\nearrow +$ N2L towards centre: $mg\cos\theta - R = \frac{mv^2}{a}$

$\Rightarrow R = mg\cos\theta - \frac{m}{a}(u^2 + 2ga(1-\cos\theta))$

$\Rightarrow R = mg\cos\theta - \frac{mu^2}{a} - 2mg + 2mg\cos\theta$

$\Rightarrow R = 3g\cos\theta - \frac{mu^2}{a} - 2mg$

$\Rightarrow R = mg(3\cos\theta - 2) - \frac{mu^2}{a}$ shown.

The particle loses contact with the sphere when $\cos\theta = \frac{3}{4}$.

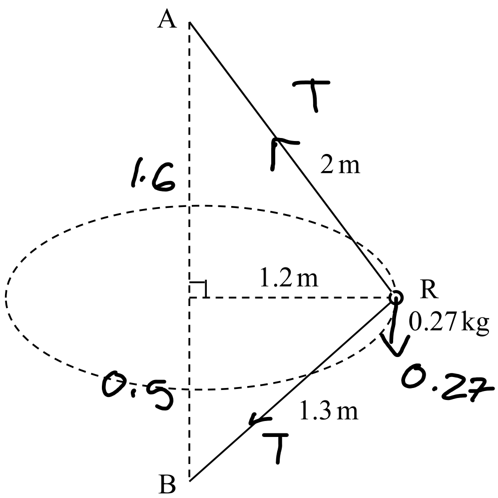
(iii) Find an expression for u in terms of a and g. [2]

@ this point $R = 0$

$\Rightarrow 0 = mg\left(\frac{9}{4} - 2\right) - \frac{mu^2}{a}$

$\Rightarrow \frac{mu^2}{a} = mg\frac{1}{4} \Rightarrow u^2 = \frac{1}{4}ag \Rightarrow u = \frac{1}{2}\sqrt{ag}$

5 Fig. 5 shows a light inextensible string of length 3.3 m passing through a small smooth ring R. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R has mass 0.27 kg and is moving with constant speed in a horizontal circle of radius 1.2 m. The distances AR and BR are 2 m and 1.3 m respectively.



$\cos\alpha = \frac{0.5}{1.3} = \frac{5}{13}$

$\cos\theta = \frac{1.6}{2} = \frac{4}{5}$

Fig. 5

(i) Show that the tension in the string is 6.37 N. [4]

Resolving vertically. $T\cos\theta = 0.27g + T\cos\alpha$

$\Rightarrow T \times \frac{4}{5} = 0.27g + T \times \frac{5}{13}$

$\Rightarrow \frac{27}{65}T = 0.27g$

$\Rightarrow T = \underline{\underline{6.37N}}$

(ii) Find the speed of R. [4]

N2L radially: $T\sin\theta + T\sin\alpha = \dfrac{0.27v^2}{1.2}$

$\Rightarrow T\left(\dfrac{1.2}{2}\right) + {}^{1}\left(\dfrac{1.2}{1.3}\right) = 0.225v^2$

$\Rightarrow 6.37\left(0.6 + \dfrac{12}{13}\right) = 0.225v^2$

$\Rightarrow v^2 = 43.12$

$\Rightarrow v = \underline{\underline{6.57\,ms^{-1}}}$

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Section B (94 marks)

Answer all the questions

6 Fig. 6 shows a pendulum which consists of a rod AB freely hinged at the end A with a weight at the end B. The pendulum is oscillating in a vertical plane. The total energy, E, of the pendulum is given by

$$E = \frac{1}{2} I \omega^2 - mgh\cos\theta,$$

where

- ω is its angular speed
- m is its mass
- h is the distance of its centre of mass from A
- θ is the angle the rod makes with the downward vertical
- g is the acceleration due to gravity
- I is a quantity known as the moment of inertia of the pendulum.

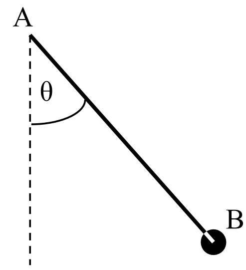

Fig. 6

(i) Use the expression for E to deduce the dimensions of I. [4]

$$ML^2T^{-2} = [I](T^{-1})^2 - M \cdot LT^{-2} \cdot L$$

$$ML^2T^{-2} = [I]T^{-2} - ML^2T^{-2}$$

$$[I]T^{-2} = ML^2T^{-2} \Rightarrow [I] = \underline{\underline{ML^2}}$$

It is suggested that the period of oscillation, T, of the pendulum is given by $T = kI^{\alpha}(mg)^{\beta}h^{\gamma}$, where k is a dimensionless constant.

(ii) Use dimensional analysis to find the values of α β and γ [5]

$$T = kI^{\alpha}(mg)^{\beta}(h)^{\gamma}$$

$$T = (ML^2)^{\alpha}(MLT^{-2})^{\beta}(L)^{\gamma}$$

$$T = M^{\alpha-\beta} L^{2\alpha+\beta+\gamma} T^{-2\beta}$$

Equating powers of T: $1 = -2\beta \Rightarrow \beta = -\frac{1}{2}$

Equating powers of M: $0 = \alpha + \beta \Rightarrow 0 = \alpha - \frac{1}{2}$

$\Rightarrow \alpha = \frac{1}{2}$

Equating powers of : $0 = 2\alpha + \beta + \gamma = 1 - \frac{1}{2} + \gamma$

$\Rightarrow \gamma = -\frac{1}{2}$ $\quad \therefore \alpha = \frac{1}{2},\ \beta = -\frac{1}{2},\ \gamma = -\frac{1}{2}$

A class experiment finds that, when all other quantities are fixed, T is proportional to $\frac{1}{\sqrt{m}}$.

(iii) Determine whether this result is consistent with your answer to part (ii). [1]

If all other quantities are fixed, then $kI^{\alpha}g^{\beta}h^{\gamma}$ is a constant. $T \propto m^{\gamma} \Rightarrow T \propto m^{-\frac{1}{2}}$

So T is proportional to $m^{-\frac{1}{2}}$ and the result is consistent with (ii).

7 A uniform ladder of length 8m and weight 180N stands on a rough horizontal surface and rests against a smooth vertical wall. The ladder makes an angle of 20° with the wall. A woman of weight 720N stands on the ladder. Fig. 7 shows this situation modelled with the woman's weight acting at a distance x m from the lower end of the ladder.

The system is in equilibrium.

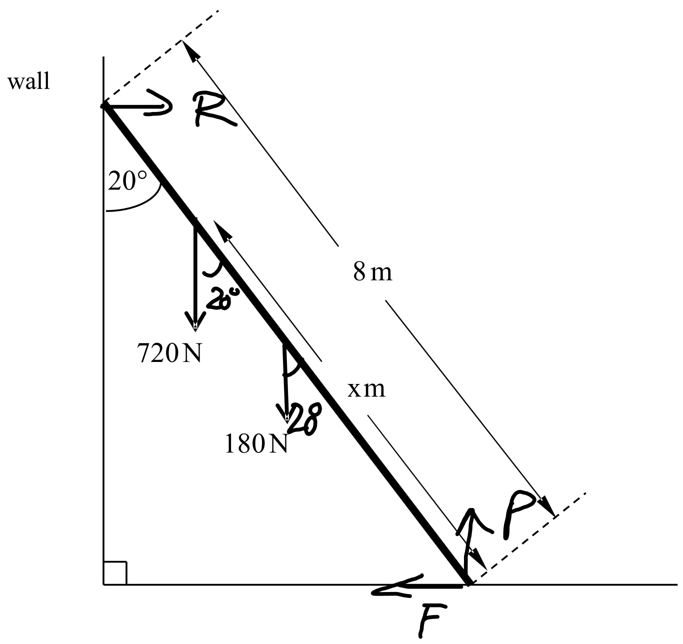

Fig. 7

(i) Show that the frictional force between the ladder and the horizontal surface is F N, where $F = 90(1 + x)\tan 20°$. [4]

Resolving horizontally: $R = F$

$\circlearrowleft$ (G) : $180\sin 20 \times 4 + 720\sin 20 \times x = R\cos 20 \times 8$

$\Rightarrow 720 \sin 20 + 720 x \sin 20 = 8R \cos 20$

$\Rightarrow 720 \sin 20 (1 + x) = 8R \cos 20$

$\Rightarrow R = \dfrac{720 \sin 20 (1+x)}{8 \cos 20}$

$\Rightarrow R = \underline{\underline{90(1+x)\tan 20}}$ shown.

(ii) (A) State with a reason whether F increases, stays constant or decreases as x increases. [1]

F increases as x increases because all other terms stay the same.

(B) Hence determine the set of values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. [4]

F is highest when $x = 8$.

$\therefore R = 90(9)\tan 20$

$= 810 \tan 20 - F$

Resolving vertically: $P = 720 + 180$

$= \underline{\underline{900\text{N}}}$

$F \leq \mu R$

$\Rightarrow 810 \tan 20 \leq \mu \times 900$

$\Rightarrow \mu \geq \dfrac{810 \tan 20}{900}$

$\Rightarrow \mu \geq \underline{\underline{0.328}}$ (3sf.)

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8 A tractor has a mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of $2.5\,\mathrm{m\,s^{-1}}$ across a horizontal field.

(i) Calculate the magnitude of the resistance to the motion of the tractor. [2]

$R \leftarrow O \rightarrow F$

$\overrightarrow{2.5\,\mathrm{ms^{-1}}}$

6000kg

$P = Fv$

$5000 = F \times 2.5$

$\Rightarrow F = 2000\text{ N}$

$F - R = 0 \Rightarrow R = \underline{\underline{2000\text{N}}}$

The tractor comes to horizontal ground where the resistance to motion is different. The power developed by the tractor during the next 10 s has an average value of 8 kW. During this time, the tractor accelerates uniformly from $2.5\,\mathrm{m\,s^{-1}}$ to $3\,\mathrm{m\,s^{-1}}$.

(ii) (A) Show that the work done against the resistance to motion during the 10 s is 71 750 J. [4]

Cons. of Energy: Initial KE + Initial PE + Work = Final KE + Final PE + W against Res.

$= \frac{1}{2}mu^2 + 0 + Pt = \frac{1}{2}mv^2 + 0 + W$

$\Rightarrow \frac{1}{2} \times 6000 \times 2.5^2 + 0 + 80000 = \frac{1}{2} \times 6000 \times 3^2 + 0 + W$

$\Rightarrow 18750 + 80000 = 27000 + W \Rightarrow W = \underline{\underline{71750\text{J}}}$

(B) Assuming that the resistance to motion is constant, calculate its value. [3]

$u = 2.5 \quad v = 3 \quad t = 10 \quad s = ?$

$s = \frac{1}{2}(u+v)t = \frac{1}{2}(5.5) \times 10 = \underline{27.5\text{ m}}$

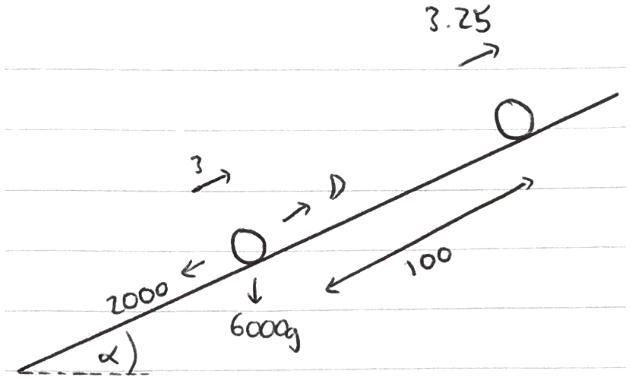
$WD = F$

$71750 = 27.5 \times F \Rightarrow F = \underline{\underline{2610\text{ N}}}$

The tractor can usually travel up a straight track inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$, while accelerating uniformly from $3\ \mathrm{m\,s^{-1}}$ to $3.25\ \mathrm{m\,s^{-1}}$ over a distance of 100 m against a resistance to motion of constant magnitude of 2000 N.

The tractor develops a fault which limits its maximum power to 16 kW.

(iii) Determine whether the tractor could now perform the same motion up the track.
[You should assume that the mass of the tractor and the resistance to motion remain the same.] [7]



$s = 100 \quad u = 3 \quad v = 3.25 \quad a = a$

$v^2 = u^2 + 2as \Rightarrow 3.25^2 = 9 + 200a$

$\Rightarrow a = \dfrac{3.25 - 3^2}{200} \Rightarrow a = \underline{0.0078125 \text{ ms}^{-2}}$

N2L $+\nearrow$: $D - 2000 - 6000g\sin\alpha = 6000 \times 0.0078125$

$D = 46.875 + 2000 + 2940$

$D = \underline{4986.875 \text{ N}}$

Max power $= Fv$

$= 4986.875 \times 3.25$

$= 16207 \text{ W}$

$= \underline{16.2 \text{ kW}}$ (3sf)

The tractor has a max power of 16kW. Therefore, it won't be able to achieve the needed power of 16.2kW.

9

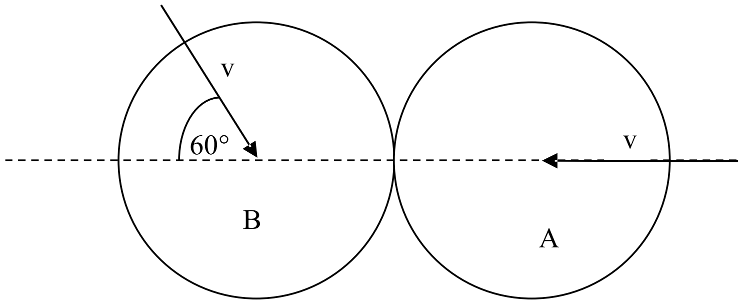


Fig. 9

Fig. 9 shows the instant of impact of two identical uniform smooth spheres, A and B, each with mass m. Immediately before they collide, the spheres are sliding towards each other on a smooth horizontal table in the directions shown in the diagram, each with speed v. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Show that, immediately after the collision, the speed of A is $\frac{1}{8}$v. Find its direction of motion. [6]

Using horizontal components of velocity.

$$\frac{V_A - V_B}{U_B - U_A} = e = \frac{1}{2}$$

$$\Rightarrow \frac{V_A - V_B}{v\cos 60 - (-v)} = \frac{1}{2} \Rightarrow V_A - V_B = \frac{1}{2}\left(\frac{1}{2}v + v\right)$$

$$= V_A - V_B = \frac{3}{4}v \rightarrow ①$$

PCLM: $v\cos 60(\cancel{m}) - v(\cancel{m}) = V_A(\cancel{m}) + V_B(\cancel{m})$

$$-\frac{1}{2}v = V_A + V_B \rightarrow ②$$

$① + ② \Rightarrow 2V_A = \frac{1}{4}v \Rightarrow V_A = \frac{1}{8}v$ (to the right, ∥ to AB)

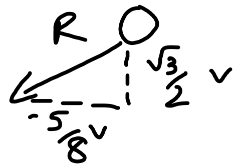
(ii) Find the percentage of the original kinetic energy that is lost in the collision. [7]

$\Rightarrow$: $V_B = V_A - \frac{3}{4}v = \frac{1}{8}v - \frac{3}{4}v = -\frac{5}{8}v$

$\uparrow$: $V_{B\uparrow}$ momentum. $v\sin 60(m) = V_{B\uparrow}(m)$

$$V_{B\uparrow} = \frac{\sqrt{3}}{2}v$$

B after.



$$R^2 = \left(-\frac{5}{8}v\right)^2 + \left(\frac{\sqrt{3}}{2}v\right)^2$$

$$R^2 = \frac{73}{64}v^2 \Rightarrow R = \frac{\sqrt{73}}{8}v$$

$$\text{Initial KE} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

$$\text{Fin KE} = \frac{1}{2}m\left(v\frac{\sqrt{73}}{8}\right)^2 + \frac{1}{2}m\left(\frac{1}{8}v\right)^2$$

$$= \frac{37}{64}mv^2$$

$$\text{Loss of KE} = \text{Initial} - \text{Final} = \frac{27}{64}mv^2$$

$$\%\text{ loss} = \frac{27}{64}mv^2 \div mv^2 = 0.422 \approx \underline{\underline{42.2\%}} \text{ (3sf)}$$

(iii) State where in your answer to part (i) you have used the assumption that the contact between the spheres is smooth. [1]

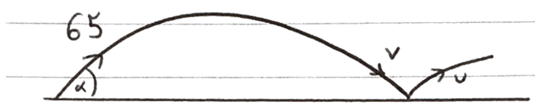
The component of linear momentum of A perpendicular to the line of centre does not change.

10 In this question take $g = 10$.

A smooth ball of mass 0.1kg is projected from a point on smooth horizontal ground with speed $65\,\text{m s}^{-1}$ at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. While it is in the air the ball is modelled as a particle moving freely under gravity. The ball bounces on the ground repeatedly. The coefficient of restitution for the first bounce is 0.4.

(i) Show that the ball leaves the ground after the first bounce with a horizontal speed of $52\,\text{m s}^{-1}$ and a vertical speed of $15.6\,\text{m s}^{-1}$. Explain your reasoning carefully. [4]

$\tan\alpha = 3/4 \Rightarrow \cos\alpha = 4/5$ & $\sin\alpha = 3/5$



$s = 0 \quad u_y = 65\sin\alpha = 39 \quad v_y = v\uparrow \quad a = -10$

$v^2 = u^2 + 2as \Rightarrow v_y^2 = 39^2 - 20(0) \Rightarrow v_y = \underline{39\ ms^{-1}}$

$v_x = v\rightarrow = 65\cos\alpha = 52\ ms^{-1}$

PCLM is conserved horizontally, $\therefore u_x = 52$

$\frac{u_y}{v_y} = 0.4 \Rightarrow u_x = 0.4 \times 39 = \underline{15.6\ ms^{-1}}$

(ii) Calculate the magnitude of the impulse exerted on the ball by the ground at the first bounce. [2]

$I = m(v - u) = 0.1(15.6 - (-39))$

$\Rightarrow I = \underline{5.46\ Ns}$

Each subsequent bounce is modelled by assuming that the coefficient of restitution is 0.4 and that the bounce takes no time. The ball is in the air for T_1 seconds between projection and bouncing the first time, T_2 seconds between the first and second bounces, and T_n seconds between the $(n-1)$th and nth bounces.

(iii) (A) Show that $T_1 = \frac{39}{5}$. [2]

$u = 39 \quad v = -39 \quad a = -10 \quad t = T_1$

$v = u + at$

$-39 = 39 - 10t$

$10t = 78$

$t = 39/5 = T_1$ <u>shown</u>.

(B) Find an expression for T_n in terms of n. [2]

The sequence is a geometric progression with

$a = \frac{39}{5}$ and $r = 0.4$.

$$T_n = \frac{39}{5} \times 0.4^{n-1}$$

(iv) According to the model, how far does the ball travel horizontally while it is still bouncing? [3]

$$S_\infty = \frac{a}{1-r} = \frac{39/5}{1-0.4} = 13$$

$$v = \frac{s}{t} \Rightarrow s = vt = 52 \times 13 = \underline{\underline{676m}}$$

(v) According to the model, what is the motion of the ball after it has stopped bouncing? [1]

Carries on with a horizontal velocity of $52ms^{-1}$.

11 The region bounded by the x-axis and the curve $y=\frac{1}{2}k\left(1-x^2\right)$ for $-1\le x\le 1$ is occupied by a uniform lamina, as shown in Fig. 11.1.

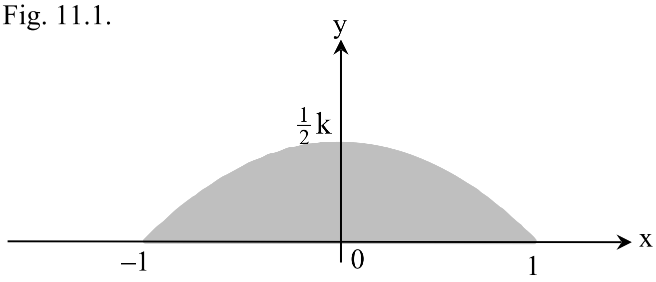


Fig. 11.1

(i) In this question you must show detailed reasoning.

Show that the centre of mass of the lamina is at $\left(0, \frac{1}{5}k\right)$. [7]

$\bar{x} = 0$ by symmetry

$\bar{y}\sigma \int_{-1}^{1} \frac{1}{2}k(1-x^2)\,dx = \sigma \int_{-1}^{1} \frac{1}{8}k^2(1-x^2)^2\,dx$

$\frac{\bar{y}}{2}k\left[x-\frac{1}{3}x^3\right]_{-1}^{1} = \frac{1}{8}k^2\left[x-\frac{2}{3}x^3+\frac{1}{5}x^5\right]_{-1}^{1}$

$\bar{y}k\left[1-\frac{1}{3}+1-\frac{1}{3}\right] = \frac{1}{4}k^2\left[1-\frac{2}{3}+\frac{1}{5}+1-\frac{2}{3}+\frac{1}{5}\right]$

$\frac{2}{3}\bar{y} = \frac{2}{15}k$

$\bar{y} = \frac{1}{5}k$

$\therefore (\bar{x}, \bar{y}) = \left(0, \frac{1}{5}k\right)$

A shop sign is modelled as a uniform lamina in the form of the lamina in part (i) attached to a rectangle ABCD, where AB = 2 and BC = 1. The sign is suspended by two vertical wires attached at A and D, as shown in Fig. 11.2.

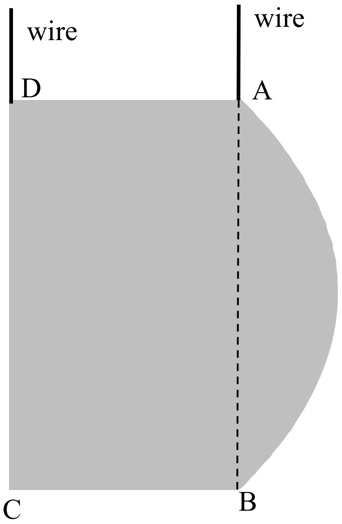


Fig. 11.2

(ii) Show that the centre of mass of the sign is at a distance

$$\frac{2k^2 + 10k + 15}{10k + 30}$$

from the midpoint of CD. [4]

Area $= \frac{2}{3}k$

Dist of C.o.M from DC $= \frac{k}{5} + 1$

Area $= 2 \times 1 = 2$

Dist of C.o.M from DC $= 0.5$

$$\bar{x}\left(\tfrac{2}{3}k + 2\right) = \tfrac{2}{3}k\left(\tfrac{1}{5}k + 1\right) + 2(0.5)$$

$$\bar{x}\left(\frac{2k+6}{3}\right) = \frac{2}{15}k^2 + \frac{2}{3}k + 1 \quad \times 15$$

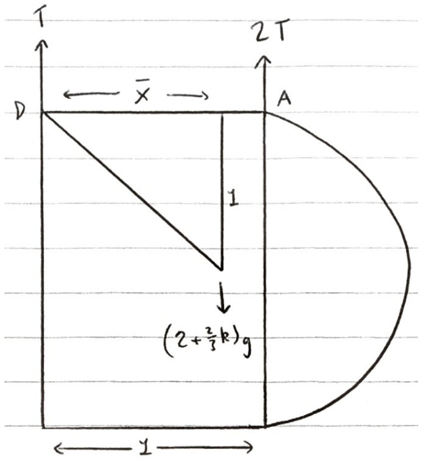
$$\bar{x}(10k + 30) = 2k^2 + 10k + 15$$

$$\bar{x} = \frac{2k^2 + 10k + 15}{10k + 30}$$

Shown

The tension in the wire at A is twice the tension in the wire at D.

(iii) Find the value of k. [5]



$$R(\uparrow):\ T + 2T = \left(2+\frac{2}{3}k\right)g$$

$$T = \frac{1}{3}\left(2+\frac{2}{3}k\right)g$$

$$\circlearrowleft D:\ 1 \times 2T = \bar{x} \times \left(2+\frac{2}{3}k\right)g$$

$$\Rightarrow\ 2T = \frac{2k^2+10k+15}{10k+30} \times \left(2+\frac{2}{3}k\right)g$$

$$\Rightarrow\ 2 \times \frac{1}{3}\cancel{\left(2+\frac{2}{3}k\right)g} = \frac{2k^2+10k+15}{10k+30} \times \cancel{\left(2+\frac{2}{3}k\right)g}$$

$$\Rightarrow\ \frac{2}{3}(10k+30) = 2k^2+10k+15$$

$$\Rightarrow\ 20k+60 = 6k^2+30k+45$$

$$\Rightarrow\ 0 = 6k^2+10k-15$$

Using calculator: $k = \dfrac{-5 \pm \sqrt{115}}{6}$

k must be '+'ve $\therefore\ k = \dfrac{-5+\sqrt{115}}{6} \approx \underline{\underline{0.954}}$ (3sf.)

12 Fig. 12 shows x- and y- coordinate axes with origin O and the trajectory of a particle projected from O with speed $28\,\text{m s}^{-1}$ at an angle α to the horizontal. After t seconds, the particle has horizontal and vertical displacements xm and ym.

Air resistance should be neglected.

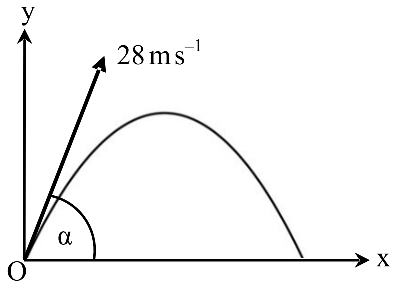


Fig. 12

(i) Show that the equation of the trajectory is given by

$$\tan^2\alpha - \frac{160}{x}\tan\alpha + \frac{160y}{x^2} + 1 = 0. \qquad (*)$$ [5]

$x = 28\cos\alpha\, t \Rightarrow t = \dfrac{x}{28\cos\alpha} \rightarrow ①$

$y = u_y t - \frac{1}{2}gt^2 = 28\sin\alpha\, t - 4.9t^2 \rightarrow ②$

Subs ① in ②:

$y = \cancel{28}\left(\dfrac{x}{\cancel{28}\cos\alpha}\right)\sin\alpha - 4.9\left(\dfrac{x}{28\cos\alpha}\right)^2$

$y = x\tan\alpha - \dfrac{x^2\sec^2\alpha}{160}$

$y = x\tan\alpha - \dfrac{x^2}{160}(1+\tan^2\alpha)$

$\times\dfrac{160}{x^2}$ $y = x\tan\alpha - \dfrac{x^2}{160} - \dfrac{x^2\tan^2\alpha}{160}$

$\dfrac{160}{x^2}y = \dfrac{160\tan\alpha}{x} - 1 - \tan^2\alpha \Rightarrow 0 = \tan^2\alpha - \dfrac{160\tan\alpha}{x} + 1 + \dfrac{160y}{x^2}$

shown.

(ii) (A) Show that if (*) is treated as an equation with $\tan\alpha$ as a variable and with x and y as constants, then (*) has two distinct real roots for $\tan\alpha$ when $y < 40 - \frac{x^2}{160}$. [3]

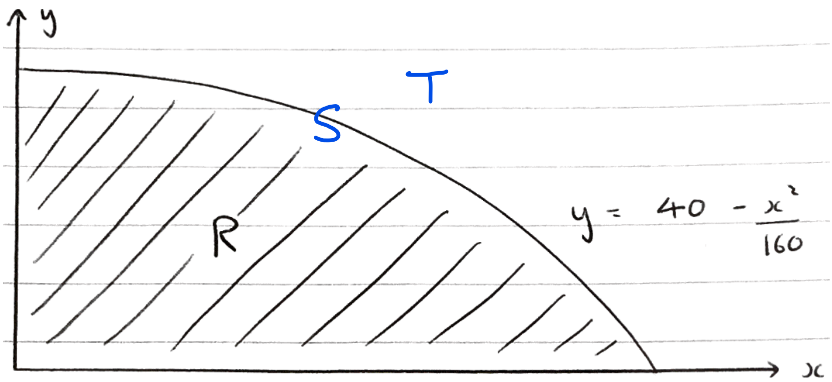
$b^2 - 4ac > 0$ for 2 distinct real roots

$\therefore \left(-\frac{160}{x}\right)^2 - 4\left(1 + \frac{160y}{x^2}\right) > 0$

$\times \frac{x^2}{4}$ $\quad \frac{160^2}{x^2} > 4 + \frac{4(160)y}{x^2}$

$\frac{160^2}{4} > x^2 + 160y \Rightarrow y < 40 - \frac{x^2}{160}$ shown.

(B) Show the inequality in part (ii) (A) as a locus on the graph of $y = 40 - \frac{x^2}{160}$ in the Printed Answer Booklet and label it R. [1]



S - on curve
T - Above Curve

S is the locus of points (x, y) where (*) has one real root for $\tan\alpha$

T is the locus of points (x, y) where (*) has no real roots for $\tan\alpha$

(iii) Indicate S and T on the graph in the Printed Answer Booklet. [2]

(iv) State the significance of R, S and T for the possible trajectories of the particle. [3]

A machine can fire a tennis ball from ground level with a maximum speed of $28\,\text{m s}^{-1}$.

R: 2 distinct roots. $\therefore$ 2 values of $\tan\alpha$, $\therefore$ 2 trajectories through each point.

T: No real roots so no trajectories through these points

S: Only one root, $\therefore$ only 1 possible trajectory through the points on curve.

(v) State, with a reason, whether a tennis ball fired from the machine can achieve a range of 80 m. [1]

Graph shows a y-value of 0 and a x value of 80, but this model doesn't consider air res., which will slow it down. So it will not reach 80m.

END OF QUESTION PAPER